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International Journal of Thermal Sciences 43 (2004) 569–574
International Journal of Thermal Sciences
www.elsevier.com/locate/ijts

Periodic free convection from vertical plate subjected to periodic surface temperature oscillation

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Received 24 March 2003; accepted 29 September 2003

Available online 7 January 2004

Abstract

The effect of the periodic oscillation of the surface temperature on the transient free convection from a vertical plate is investigated in the present paper. The problem has been simplified by the laminar boundary layer and Boussinesq approximations. The fully implicit finite-difference scheme is used to solve the dimensionless system of the governing equations. The results for laminar flow of air $Pr = 0.72$ and water $Pr = 7.00$ are presented for an isothermal flat plate and for a periodic oscillation of the plate temperature. The results presented to show the steady periodic variation of Nusselt number with the amplitude and frequency of the oscillating surface temperature. It is found that increasing the amplitude and the frequency of the oscillating surface temperature will decrease the free convection heat transfer from the plate to both air and water.

Keywords: Free convection; Transient convection; Boundary layer; Oscillating temperature; Numerical study

1. Introduction

Transient laminar free convection from vertical wall is important in many practical applications, such as furnaces, electronic components, solar collectors, chemical processing equipments and others. The change in the wall temperature causing the free convection flow could be a sudden or a periodic one, leading to a variation in the flow. If the wall surface temperature is suddenly changed from the ambient temperature to a specific value, the steady free convection flow is reached, following a transient flow which occurs for a certain period of time. The literature shows that the transient free convection problem has been investigated by various researchers. Numerical solutions of the governing boundary layer equations have been obtained by Hellums and Churchill [1] with a vertical surface subjected to a step change in the surface temperature. Hellums and Churchill [1] observed an initial undershoot of the local Nusselt number below the steady-state solution and the eventual approach to the steady flow solution. This undershoot phenomenon is observed also by Callahan and Marner [2] in both Nusselt and Sherwood numbers in the transient free convection with mass transfer on an isothermal vertical flat plate. The transient free convection from vertical flat plate is also investigated, by Harris et al. [3], when the plate temperature is suddenly changed from T_1 to T_2. They obtained an analytical solution for small values of the non-dimensional time, and detailed numerical solution of the full boundary layer equations form transient solution until the steady-state is reached.

For a non-isothermal vertical plate, with the surface temperature variation given by the power law dependence, the similarity solution introduced by Sparrow and Gregg [4] is usually used. The free convection from non-isothermal vertical wall, but with streamwise surface temperature oscillation has received attention by many researchers including Rees [5], and Li et al. [6]. In this class of free convection problems, Rees [5] has studied the effect of the sinusoidal streamwise surface temperature variation on the steady free convective boundary layer flow. He used combined numerical and asymptotic analysis to find that the rate of heat transfer will eventually alternate in sign with distance from the leading edge. The steady and unsteady free convection from vertical wall with streamwise surface temperature oscillation has been investigated by Li et al. [6]. For small values of Grashof number they obtained an asymptotic formula for the average Nusselt number using a perturbation method.

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1290-0729/$ – see front matter
doi:10.1016/j.ijthermalsci.2003.09.007

Nomenclature

$A(Nu)$	amplitude of $Nu_x/Gr_x^{1/4}$	
$f'(\eta)$	non-dimensional velocity	
g	gravitational acceleration	$m \cdot s^{-2}$
Gr	Grashof number based on L	
Gr_x	local Grashof number	
L	wall height	m
Nu_x	local Nusselt number	
$\overline{Nu}$	average of $Nu_x/Gr_x^{1/4}$	
Pr	Prandtl number	
t	time	s
T	temperature	K
u, v	velocity components	$m \cdot s^{-1}$
U, V	non-dimensional velocity components	
x, y	Cartesian coordinates	m
X, Y	non-dimensional Cartesian coordinates	

Greek symbols

α	thermal diffusivity	$m^2 \cdot s^{-1}$
β	coefficient of volume expansion	K^{-1}
η	non-dimensional similarity variable	
ε	non-dimensional amplitude	
ν	kinematic viscosity	$m^2 \cdot s^{-1}$
θ	non-dimensional temperature	
τ	non-dimensional time	
ω	frequency	s^{-1}
Ω	non-dimensional frequency	

Subscripts

w	wall
∞	ambient

It is noted that the isothermal or a streamwise plate temperature variation is usually used in the above free convection problems. But in the industrial applications, quite often the free convection is a periodic process. Lorenzo and Padet [7] have conducted a parametric study on the free convection along vertical wall when a periodic heat flux density which contains adiabatic period is applied. Their results show the importance of the adiabatic period to let the surrounding fluid to refresh itself before applying a new heating period leading to increase the free convection heat transfer. Another practical free convection problem is the periodic oscillation of the surface temperature. This problem has been addressed by Das et al. [8]. They simplify the problem by assuming small values of Grashof number Gr ($\ll 1$). In this case the temperature is independent of the flow and the heat is transferred by conduction only. Das et al. [8] have used Laplace transform technique to solve the simplified equations, and the results show that the transient velocity profile and the penetration distance deceases with increasing the frequency of the plate temperature oscillation. In the present paper, relatively higher Grashof number is considered ($10^4 < Gr < 10^9$) where the laminar boundary layer theory is applicable to study the effect of periodic plate temperature oscillation on the free convection from vertical plate.

2. Analysis

The governing equations of the present problem are the continuity, momentum and energy. They are coupled elliptic equations, and therefore of considerable complexity. To overcome such difficulty, the boundary layer and Boussinesq approximations [9] are used in the present study. The resulting two-dimensional equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where the vertical wall is considered to be along x-axis and y-axis is normal to it. In accordance with the present problem, the initial and boundary conditions are:

$$u(x, y, 0) = v(x, y, 0) = 0, \qquad T(x, y, 0) = T_\infty \tag{4a}$$

$$u(0, y, t) = v(0, y, t) = 0, \qquad T(0, y, t) = T_\infty \tag{4b}$$

$$u(x, 0, t) = v(x, 0, t) = 0, \qquad T(x, 0, t) = T_w(t) \tag{4c}$$

$$u(x, \infty, t) = 0, \qquad T(x, \infty, t) = T_\infty \tag{4d}$$

The wall temperature condition is assumed to oscillate periodically over an average value $\overline{T}_w$ with amplitude ε and frequency ω. Therefore the following wall temperature condition is used:

$$T_w(t) = \overline{T}_w + \varepsilon(\overline{T}_w - T_\infty)\sin\omega t \tag{5}$$

In order to simplify the problem and to generalize the results for $10^4 < Gr < 10^9$, the above equations are written in a non-dimensional form by employing the following boundary layer dimensionless variables:

$$\begin{aligned} &U = u/u_c, \qquad V = \frac{v}{u_c}Gr^{1/4} \\ &X = x/L, \qquad Y = \frac{y}{L}Gr^{1/4} \\ &\tau = t/t_c, \qquad \Omega = \omega t_c, \qquad \theta = \frac{T - T_\infty}{\overline{T}_w - T_\infty} \end{aligned} \tag{6}$$

where u_c, t_c are the characteristic velocity and time scales respectively, L is the wall height and $Gr = g\beta L^3 \Delta T/\nu^2$ is the Grashof number based on the characteristic length L and average temperature difference $\Delta T = \overline{T}_w - T_\infty$. Using (6), the momentum and energy equations become:

$$\frac{u_c}{t_c g\beta\Delta T}\frac{\partial U}{\partial\tau}+\frac{u_c^2}{g\beta L\Delta T}\left(U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}\right)$$
$$=\frac{\nu u_c\sqrt{Gr}}{g\beta L^2\Delta T}\frac{\partial^2 U}{\partial Y^2}+\theta \qquad (7)$$

$$\frac{\partial\theta}{\partial\tau}+\frac{t_c u_c}{L}\left(U\frac{\partial\theta}{\partial X}+V\frac{\partial\theta}{\partial Y}\right)=\frac{\alpha t_c\sqrt{Gr}}{L^2}\frac{\partial^2\theta}{\partial Y^2} \qquad (8)$$

The characteristic velocity is usually taken as the maximum velocity generated in the flow [9], which is defined as:

$$u_c=\sqrt{g\beta L\Delta T} \qquad (9)$$

which is used in the present analysis together with the following characteristic time scale:

$$t_c=\sqrt{L/(g\beta\Delta T)} \qquad (10)$$

Then the governing equations reduce to the following non-dimensional boundary layer equations:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \qquad (11)$$

$$\frac{\partial U}{\partial\tau}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\frac{\partial^2 U}{\partial Y^2}+\theta \qquad (12)$$

$$\frac{\partial\theta}{\partial\tau}+U\frac{\partial\theta}{\partial X}+V\frac{\partial\theta}{\partial Y}=\frac{1}{Pr}\frac{\partial^2\theta}{\partial Y^2} \qquad (13)$$

where $Pr=\nu/\alpha$ is the Prandtl number. The dimensionless initial and boundary conditions (4) become:

$$U(X,Y,0)=V(X,Y,0)=0,\qquad \theta(X,Y,0)=0 \qquad (14a)$$

$$U(0,Y,\tau)=V(0,Y,\tau)=0,\qquad \theta(0,Y,\tau)=0 \qquad (14b)$$

$$U(X,0,\tau)=V(X,0,\tau)=0,$$
$$\theta(X,0,\tau)=\theta_w(\tau)=1+\varepsilon\sin(\Omega\tau) \qquad (14c)$$

$$U(X,\infty,\tau)=0,\qquad \theta(X,\infty,\tau)=0 \qquad (14d)$$

No analytical solution is known for Eqs. (11)–(13) with the initial and boundary conditions (14). In the steady-state case, the similarity solution is usually used and the system of Eqs. (11)–(13) can be transferred to a system of coupled ordinary differential equations which can be solved using shooting methods. The Cartesian coordinates (x, y) have been used in solving unsteady free convection problems in pure viscous fluids [1,2,6,8,10] as well as in the porous media [11,12]. Therefore, the present periodic free convection problem is solved numerically using the Cartesian coordinates.

3. Numerical scheme

The momentum and energy equations (12) and (13) are integrated over a control volume using the fully implicit scheme which is unconditionally stable. The power-law scheme is used for the convection–diffusion formulation [13]. Finally, the finite-difference equation corresponding to the continuity equation (11) is developed using the expansion point $(i+1, j-\frac{1}{2})$, where i and j are the indices along X and Y, respectively, [14]. The resulting finite-difference equation is:

$$V_{i+1,j}=V_{i+1,j-1}-\frac{\Delta Y_w}{2(\Delta X_n)}\times(U_{i+1,j}+U_{i+1,j-1}-U_{i,j}-U_{i,j-1}) \qquad (15)$$

where ΔY_w and ΔX_n are the grid spaces west and north of the (i, j) point, respectively. The solution domain, therefore, consists of grid points at which the discretization equations are applied. In this domain X by definition varies from 0 to 1. But the choice of the value of Y, corresponding to $Y=\infty$, has an important influence on the solution. The effect of different values to represent $Y=\infty$ on the numerical scheme has been investigated and it is concluded that the value of $Y=10$ is sufficiently large. Further larger values of Y produced the results with indistinguishable difference. The stretched grid has been selected in both X and Y direction such that the grid points clustered near the wall and near the leading edge of the flat plate as there are steep variation of the velocities and temperatures in these regions.

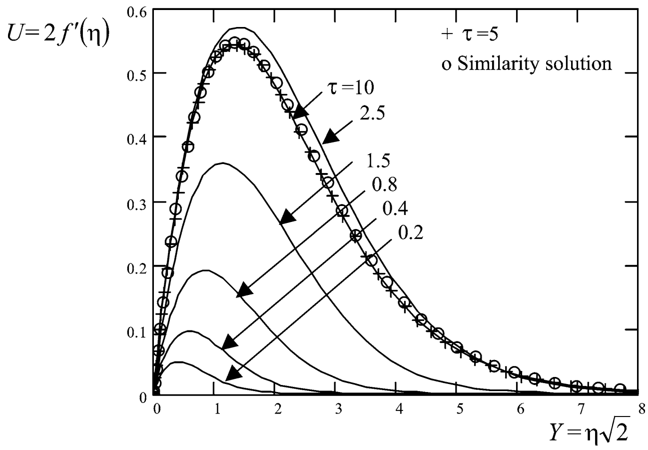
The algorithm needs iteration for the coupled equations (11)–(13). First iteration starts to solve the discretized energy equation from zero initial velocities and temperatures in all the grid points using line-by-line tridiagonal-matrix algorithm. This means that Eq. (13) is reduced to the transient heat conduction equation. The resultant temperature field is then used to solve the discretized momentum equation to find U profiles from the zero V values. Finally the V profiles are found from the solution of the discretized continuity equation (15) explicitly. The iteration then continues to solve for θ, U, and V using the pervious iteration values until iterative converged solution is obtained. The convergence condition used for the three dependent variables θ, U, and V is:

$$\text{Max}\left|\frac{\varphi^n-\varphi^{n-1}}{\varphi^n}\right|<10^{-5} \qquad (16)$$

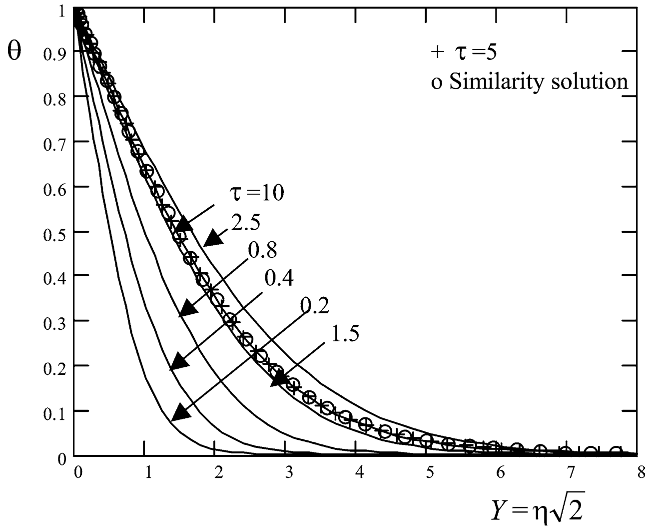
where φ is the general dependent variable and the superscript n represents the iteration step number. The time increment is $\Delta\tau=0.01$ for the isothermal wall case, and $\Delta\tau=2\pi/(1000\Omega)$ for the oscillation surface temperature case. It is found that smaller than these values have no significant difference in the results.

4. Results and discussion

The algorithm explained in the pervious section is first tested before studying the effect of surface temperature oscillation on the transient free convection from vertical plate. The test is selected to study the laminar transient free convection with a step change in the surface temperature to the average value $\overline{T}_w$ with $Pr=0.72$, i.e., $\varepsilon=0$ in Eq. (14c). At the initial stage of the transient, the transport mechanism is predominantly conduction. On the other hand at larger time, the flow will be at steady-state condition. Therefore



(a)



(b)

Fig. 1. Transient velocity and temperature profiles for air ($Pr = 0.72$): (a) Velocity profiles; (b) Temperature profiles.

the transient results are expected to fall within these two limits. Several time steps are chosen for calculations to demonstrate this fact, which are $\tau = 0.2, 0.4, 0.8, 1.5, 2.5, 5$ and 10. Fig. 1 shows the velocity and temperature profiles, at the upper end of the wall, plotted at different time steps until the steady-state is reached ($\tau \geqslant 5$). The steady-state velocity and temperature profiles obtained from similarity solution [15] are also presented in Fig. 1 for comparison. It is important to note that in the similarity solution [15], the non-dimensional velocity is defined as $f'(\eta) = u/(2\sqrt{x\beta g\Delta T})$ and $\eta = (y/x)(Gr_x/4)^{1/4}$, while the definition of the non-dimensional temperature is same as in (6). It can be shown that $U = 2f'(\eta)$ and $Y = \eta\sqrt{2}$ at the upper end of the plate where $X = 1$. An excellent agreement of the present results with the similarity results is shown in Fig. 1 for both velocity and temperature profiles. It is observed that the dimensionless velocity and temperature, at the upper end of the wall, increase with time to reach maximum value and then decrease to reach the steady-state values. Thus the heat transfer coefficient and hence, the local Nusselt number will decrease with time and it reaches a minimum value and then increase slightly to approach the steady-state value as shown in Fig. 2 for $Pr = 0.72$ (air) and $Pr = 7.00$ (water). The transient local Nusselt number is defined in the present study as:

$$Nu_x = \frac{-k[\mathrm{d}T/\mathrm{d}y]_{y=0}}{T_w(t) - T_\infty}\frac{x}{k} \tag{17}$$

or,

$$\frac{Nu_x}{Gr_x^{1/4}} = \frac{-X^{1/4}}{[\theta_w(\tau)]^{5/4}}\left[\frac{\mathrm{d}\theta}{\mathrm{d}Y}\right]_{Y=0} \tag{18}$$

where k is the thermal conductivity of the fluid, and $Gr_x = g\beta x^3\{T_w(t) - T_\infty\}/\nu^2$ is the local Grashof number. The results for all the time steps considered and, the results of Hellums and Churchill [1] are presented in Fig. 2 for comparison. A good agreement is observed including the initial undershoot below the steady-state solution for all the time steps. Fig. 2 shows that the ratio $Nu_x/Gr_x^{1/4}$ for water is higher than that of air, and higher values of Pr take longer time to reach the steady-state. These results provided confidence to the accuracy of the present numerical model to study the effect of the periodic oscillation of the wall surface on the transient free convection from a vertical flat plate.

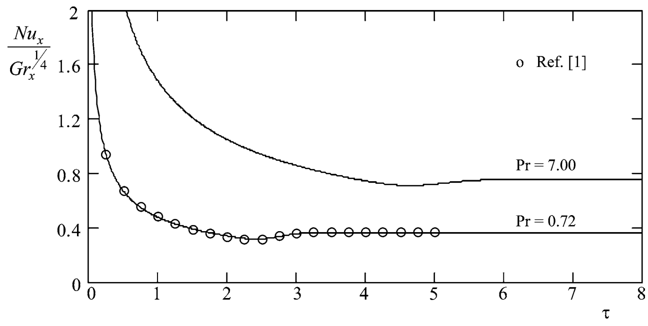


Fig. 2. Variation of the local Nusselt number with τ for air ($Pr = 0.72$) and water ($Pr = 7.00$).

The oscillation of the wall temperature is considered now with amplitude range from $\varepsilon = 0.1$ to $\varepsilon = 0.5$ and frequency range from $\Omega = 0.25$ to $\Omega = 5$. The free convection process starts when the surface temperature increases suddenly from the ambient temperature T_∞ to average surface temperature $\overline{T}_w$. At this time the ratio $Nu_x/Gr_x^{1/4}$ goes to infinity. Then, when the surface temperature oscillates the ratio $Nu_x/Gr_x^{1/4}$ is found to oscillate accordingly. This oscillation becomes steady periodic oscillation after some periods. The steady periodic oscillation is achieved when the amplitude and the average values of the ratio $Nu_x/Gr_x^{1/4}$ becomes constant for different periods. The following condition is considered for the steady periodic oscillation:

$$\frac{A(Nu)^p - A(Nu)^{p-1}}{A(Nu)^p} \leqslant 10^{-5} \quad \text{and}$$
$$\frac{\overline{Nu}^p - \overline{Nu}^{p-1}}{\overline{Nu}^p} \leqslant 10^{-5} \tag{19}$$

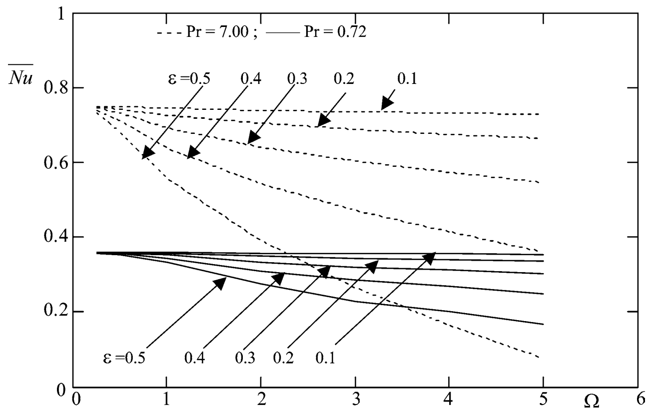


Fig. 3. Variation of the average Nusselt number with the frequency at different wall temperature amplitude in the ultimate cycle for air ($Pr = 0.72$) and water ($Pr = 7.00$).

where the superscript p is the period number, and

$$A(Nu) = \frac{1}{2}\left[\mathrm{Max}\left(\frac{Nu_x}{Gr_x^{1/4}}\right) - \mathrm{Min}\left(\frac{Nu_x}{Gr_x^{1/4}}\right)\right] \tag{20}$$

for $\tau_0 \leqslant \tau \leqslant \tau_0 + (2\pi/\Omega)$ and

$$\overline{Nu} = \frac{1}{(2\pi/\Omega)} \int_{\tau_0}^{\tau_0+(2\pi/\Omega)} \left(\frac{Nu_x}{Gr_x^{1/4}}\right) \mathrm{d}\tau \tag{21}$$

Fig. 3 show the variation of the average value of the ratio $Nu_x/Gr_x^{1/4}$ designed as $\overline{Nu}$ with the non-dimensional frequency Ω, at different amplitudes of the surface temperature oscillation ε. For $\varepsilon = 0$ (isothermal wall), the steady-state average value is found $Nu_x/Gr_x^{1/4} = 0.3582$ for $Pr = 0.72$ (White [15] reported a value of 0.3568 for this isothermal wall case based on the similarity solution) and $Nu_x/Gr_x^{1/4} = 0.7479$ for $Pr = 7.00$. These average values decrease when the surface temperature oscillates for both air and water as shown in Fig. 3. When the surface temperature oscillates at high amplitude and frequency, the ratio $Nu_x/Gr_x^{1/4}$ becomes negative for some instances of the oscillation period. It is observed from Fig. 3 that for the cases when $\Omega > 3.5$ and $\varepsilon = 0.5$ the average Nusselt number for the steady periodic state for water is less than that of air, which is an unusual behavior. The steady periodic oscillation of $Nu_x/Gr_x^{1/4}$ for air and water are shown in Figs. 4(a) and (b), respectively, at different values of ε. The oscillation of Nusselt number is more intensive for high values of ε and Pr. The effect of the frequency of the surface temperature oscillation on the Nusselt number oscillation is shown in Fig. 5 for the ultimate steady period $\Omega\tau$. The Nusellt number oscillates with low amplitude for small Ω and Pr as shown in Fig. 5 and visa versa. A detail investigation has been carried out for the special behavior case ($\varepsilon = 0.5$ and $\Omega = 5$). The period of the last cycle is divided in to eight time steps (a)–(h) as shown in Fig. 4. At each time step the temperature profiles are shown in Figs. 6 and 7 for air and water respectively. It can be seen from Fig. 6 that the surface temperature is equal at the

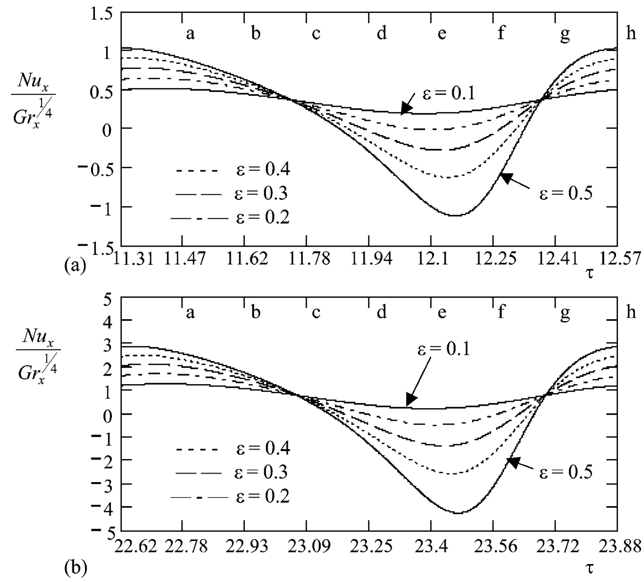


Fig. 4. Ultimate periodic oscillation of $Nu_x/Gr_x^{1/4}$ with τ at different wall temperature amplitude with $\Omega = 5$: (a) $Pr = 0.72$; (b) $Pr = 7.00$.

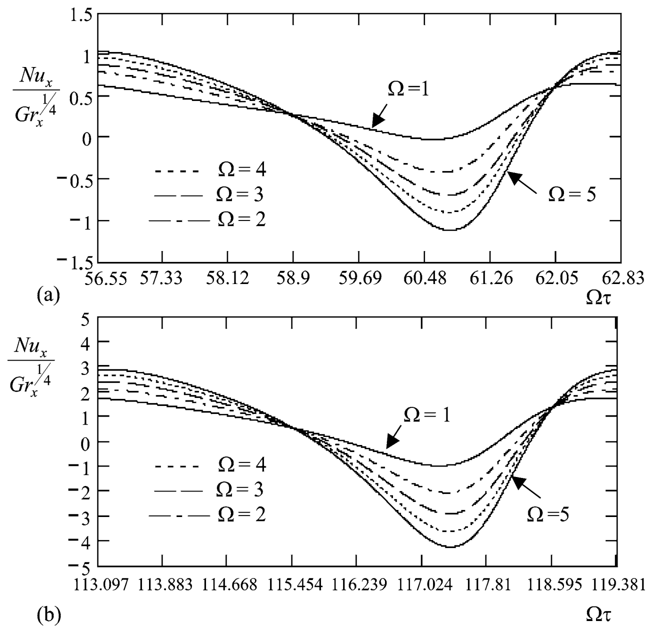


Fig. 5. Ultimate periodic oscillation of $Nu_x/Gr_x^{1/4}$ with $\Omega\tau$ at different wall temperature frequencies with $\varepsilon = 0.5$: (a) $Pr = 0.72$; (b) $Pr = 7.00$.

points (a, c), (d, h) and (e, g) but the temperature gradients are these points are different which gives different values of the Nuseelt number. The maximum value of $Nu_x/Gr_x^{1/4}$ occurs at point (h) when the surface temperature increases from its minimum value to its average value. The minimum value of $Nu_x/Gr_x^{1/4}$ is negative and it occurs between points (e) and (f) when the surface temperature goes to its minimum value. Negative values of $Nu_x/Gr_x^{1/4}$ means that there will be some point in the boundary layer with temperature higher than the surface temperature from which heat will transfer partly to the wall, or some of the heat gained by the fluid

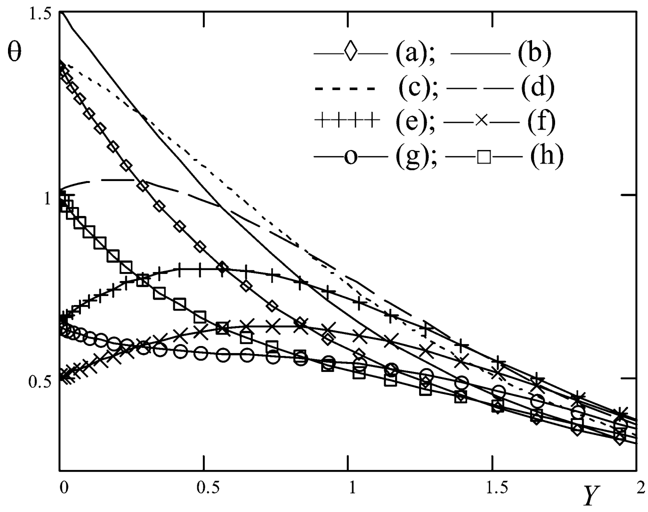


Fig. 6. Periodic state temperature profiles at different time steps ((a)–(h) in Fig. 4) for last cycle, $\varepsilon = 0.5$ and $\Omega = 5$ for $Pr = 0.72$.

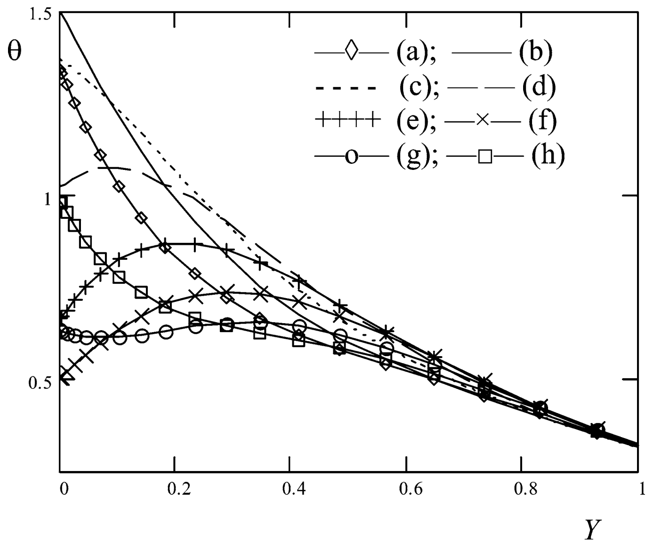


Fig. 7. Periodic state temperature profiles at different time steps ((a)–(h) in Fig. 4) for last cycle, $\varepsilon = 0.5$ and $\Omega = 5$ for $Pr = 7.00$.

in early stages will return back to it. A similar phenomenon is observed for the water with more intensive oscillation of $Nu_x/Gr_x^{1/4}$ as shown in Fig. 7. It has been also observed from Figs. 6 and 7 that the boundary layer thickness for the air is more than that for water for the surface temperature oscillation as well as in the isothermal case.

5. Conclusions

In this paper, the effect of the periodic oscillation of the surface temperature on the periodic free convection from a vertical wall is considered. In the mathematical formulation of this problem the boundary layer and Boussinesq approximations are used to simplify the problem. The dimensionless forms of the governing equations are solved numerically using finite-difference method. The developed algorithm is tested and the results are compared with the published data of the steady-state and transient free convection with fixed surface temperature. Good agreement was found. It has been shown that the ratio of the local Nusselt number to the local Grashof number oscillates as a result of oscillating surface temperature with a small phase shift. This oscillation is due to the oscillation of the temperature gradient at the wall and the oscillation of the transient temperature difference between the wall and the ambient. The results also show that the wall heat transfer decreases with increasing the amplitude and frequency of the oscillating surface temperature for both air and water.

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